

# Comparison of Attitude and Heading Reference Systems using Foot Mounted MIMU Sensor Data: Basic, Madgwick and Mahony

Simone A. Ludwig<sup>a</sup> and Kaleb D. Burnham<sup>a</sup> and Antonio R. Jiménez<sup>b</sup> and Pierre A. Touma<sup>c</sup>

<sup>a</sup>North Dakota State University, Fargo, ND, USA

<sup>b</sup>Centre for Automation and Robotics (CAR-CSIC/UPM), Arganda del Rey, Madrid, Spain

<sup>c</sup>Axelo Inc., Austin, TX, USA

## ABSTRACT

A magnetic and inertial measurement unit (MIMU) usually measures acceleration, rotation rate, and earth's magnetic field in order to determine a body's attitude. In order to find the orientation information using all sensors a fusion algorithm is used. This paper compares two approaches used for a Attitude and Heading Reference System (AHRS), namely Madgwick and Mahony with a basic fusion approach. Foot mounted MIMU data is used to estimate the Euler angles as well as the position. The results show that Madgwick obtains better heading orientation than Mahony and the basic AHRS approach in terms of the error (RMSE) of the Euler angles when compared to the ground truth. However, the execution time of Mahony is less than Madgwick with the basic AHRS taking the longest.

**Keywords:** magnetic and inertial measurement unit, fusion algorithms, foot-mounted sensor data

## 1. INTRODUCTION

Research around localization of objects and people has received significant attention in the past few decades and numerous techniques have been proposed to achieve high-accuracy localization. Although GPS is the most common technology to provide accurate location for outdoor environments, it suffers from signal blocking and the multipath effect. These effects lead to significant low location accuracy during the localization process. Thus, other alternative sensor measuring techniques have to be used and improved in order to cover or replace GPS technology.

A magnetic and inertial measurement unit (MIMU) consists of a 3-axis MEMS gyroscope, accelerometer and magnetometer. MIMUs are widely used in many applications of attitude determination such as human motion tracking, unmanned aerial vehicle (UAV), mobile navigation, etc.<sup>1</sup> The gyroscope measures the angular rate of a moving object, the accelerometer measures the acceleration of a certain object, and the magnetometer measures the magnetic field. However, low-cost sensors have inherent drawbacks,<sup>2,3</sup> such as nonlinearity, random walk, temperature drift, etc. In order to obtain a reliable attitude solution, MIMU sensor measurements have to be fused together using optimal sensor fusion algorithms.<sup>4</sup> There are mainly two different fusion approaches. One category includes the complementary filters and the other relates to Kalman filtering.

The sensor data obtained from the gyroscope and the magnetometer has been used to obtain the heading.<sup>5,6</sup> Basically, the integration of the gyroscope from a known initial orientation supplies the change in rotation. However, the gyroscope has a long-term drift which is due to noise and bias. Thus, these errors need to be corrected. The calibrated magnetometer is used to minimize the drift in the horizontal orientation. Similar work has been done previously by Muñoz et al.<sup>7</sup> Their paper describes a comparison of four different AHRS algorithms, namely the basic AHRS, Madgwick, Mahony, and a DLR-AHRS approach. The focus of their paper, however, was on the influence of magnetic disturbances.

This paper compares Madgwick and Mahony with the basic AHRS approach to estimate the Euler angles and position. Foot mounted error-free MIMU data is used to evaluate the approaches. RMSE (Root Mean Square Error) is calculated based on the ground truth and the execution time of all AHRS is measured.

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Further author information: (Send correspondence to Simone A. Ludwig): E-mail: simone.ludwig@ndsu.edu

## 2. APPROACHES

Using a MIMU we can obtain measurements for acceleration, ambient geomagnetic field and angular velocity from the accelerometer, magnetometer and gyroscope, respectively. In order to compute the orientation estimation, the measurement errors of these sensors have to be considered and reduced. Using the three sensors, it is possible to build an AHRS (Attitude and Heading Reference System) in which each sensor will compensate for the bias introduced by the others. Therefore, the accelerometer and magnetometer data has to be merged with angular velocity from the gyroscope to compute an estimate of the orientation angles. This estimate is provided by orientation filters. Examples of orientation filters are the complementary filter,<sup>8</sup> and the Kalman filter.<sup>9</sup> Examples of general purpose Bayesian estimation filters are Mahony,<sup>10</sup> and Madgwick.<sup>11</sup> Either of these filters can be used in an AHRS.

The task of an AHRS is to provide the orientation of the sensors with respect to a navigational frame. The orientation is commonly represented by Euler angles in the form of roll, pitch and yaw. Basically, the AHRS integrates the gyroscope reading subject to some absolute correction that results from the magnetic field and gravity. Thus, together with the acceleration, the roll and pitch can be obtained. Furthermore, the yaw can be obtained via the magnetometer reading.

The methods of Madgwick and Mahony differ with regards to the resolution of these biases. For example, Mahony uses a proportional and integral controller to correct the gyroscope bias, whereas Madgwick uses only a proportional controller. Both approaches use a quaternion representation, which is a four-dimensional complex number representing the orientation of an object. Even though quaternions are easier to calculate and more efficient, however, they are less intuitive and understandable than Euler angles. Euler angles on the other hand are subject to ambiguity and gimbal lock, which are two known problems of this representation that have been taken into account. Gimbal lock is a singularity that appears when two axes of the object have parallel orientation and causes the loss of one degree of freedom, and therefore measurement inaccuracy. Thus, in this paper we are comparing Madgwick and Mahony applied to foot mounted MIMU sensor data with ground truth provided in order to observe the errors involved.

The basic AHRS approach is described in details by Muñoz et al.<sup>7</sup> It consists of two components that are fused together as a weighted approach. The first part estimates the orientation from the gyroscope, and the second component provides the orientation from the accelerometer and the magnetometer. Both components provide an independent orientation estimate, and thus, are fused together in order to benefit from each source of information.

## 3. EVALUATION

The data used is foot mounted MIMU measurement data.<sup>12</sup> It contains sensor data of a straight trajectory of 1,000 steps based on a human step pattern characteristics measured by a motion capture system. For our experiments, we have only used partial data of the data set. The information from the MIMU is the acceleration, turn rates from the gyroscope and the magnetic field. The data set includes the orientation (Euler and DCM) ground truth values. The units are in meters, seconds and radians, a sampling frequency of 100 Hz was used, and gravity is  $9.8 \frac{m}{s^2}$ . The MIMU used was the XSense MTi with the following specification:

- Accelerometer:  $0.012 \frac{m}{s^2}$  standard deviation random noise and a random constant with a Gaussian distribution and a standard deviation of  $0.04 \frac{m}{s^2}$  for the bias.
- Gyroscope:  $0.0087 \frac{rad}{s}$  standard deviation random noise and a random constant with a Gaussian distribution and a standard deviation of  $0.015 \frac{rad}{s}$  for the bias.

XSense MIMU are commonly used in motion sensing applications, and are seen as the gold standard for scientific research.<sup>13–16</sup>

Figure 1 shows the sensor data obtained by the gyroscope, accelerometer, and magnetometer. The cyclic steps of the walking motion can be observed.

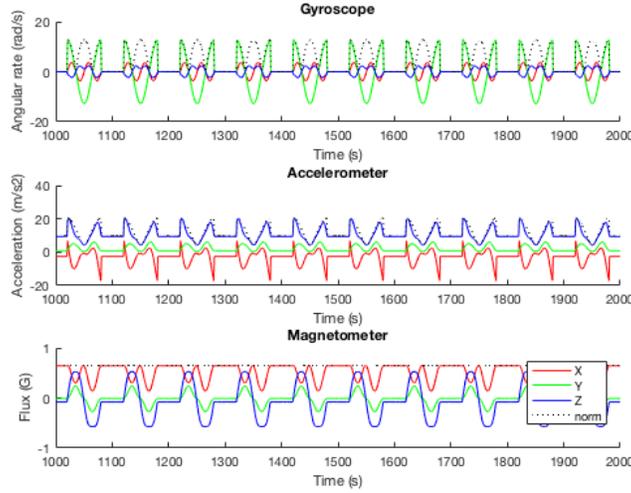


Figure 1. Sensor data of gyroscope, accelerometer, magnetometer

In Figure 2, the Euler estimation correctness obtained by the basic AHRS is shown. Furthermore, the Euler estimation correctness obtained by Madgwick is shown in Figure 3, and the one for Mahony in Figure 4. Preliminary experiments identified the optimal user-defined gains for each filter. The gain in the Madgwick filter represents all mean zero gyroscope measurement errors and the optimum value was identified by Madgwick.<sup>11</sup> In both, the Mahony and our basic filter, the gains are used as weights. The Mahony filter takes into consideration the disparity between the orientation from the gyroscope and the estimation from the magnetometer and accelerometer and weighs them according to its gains. The changes made to the gyroscope are given by:

$$k_p * e_m + k_i * e_i \quad (1)$$

where  $k_p$  is the proportional gain,  $e_m$  is the sensor error of the gyroscope,  $k_i$  is the integral gain,  $e_i$  is the integral error, which is calculated by:

$$e_m * \frac{1}{f_s} \quad (2)$$

where  $f_s$  is the sampling frequency.

The basic filter uses a simpler weighting process directly on the quaternions. It is given by:

$$\gamma * q_G + (1 - \gamma) * q_{AM} \quad (3)$$

where  $\gamma$  is the weighing coefficient,  $q_G$  is the quaternion of the gyroscope, and  $q_{AM}$  is the quaternion based on the acceleration and magnetometer readings.

We identified the optimum values for both the Mahony and the basic filter based on preliminary experiments. The parameters used for the experiments were:

- $\beta = 0.033^{11}$  (Madgwick)
- $k_p = 0.13, k_i = 0.3$  (Mahony)
- $\gamma = 0.995$  (Basic AHRS)

Table 1 shows the root mean squared error (RMSE) of the euler angles for Madgwick, Mahony and the basic AHRS. The first number represents the euler angle for the sensor data with noise and the second number

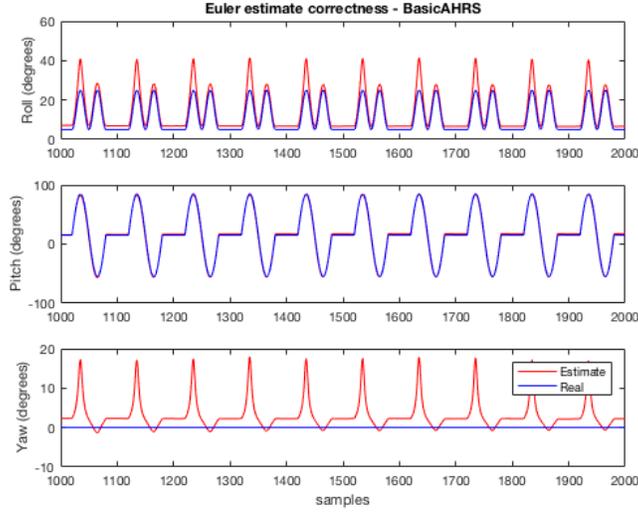


Figure 2. Euler estimation correctness of Basic AHRS

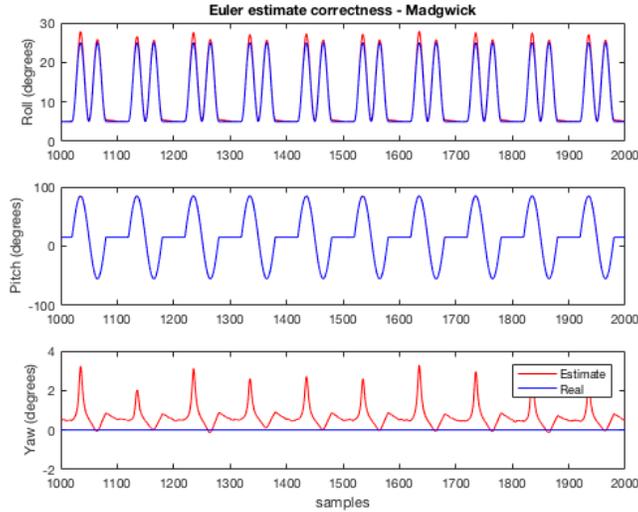


Figure 3. Euler estimation correctness of Madgwick

represents the euler angles without noise. What can be observed is that Madgwick scores best for the sensor data with noise included outperforming both Mahony and the basic AHRS. The RMSE values for the euler angles for Madgwick for the sensor data with noise are 0.5896, 0.3123, and 0.5787 for the roll, pitch and yaw, accordingly.

Figures 5, 6, and 7 show the position estimation of the 3 axes. Similarly to the euler angles, the position estimation in x, y, and z direction are the smallest when Madgwick is applied to the foot mounted data. Figure 6 shows the smallest error in all directions when compared to Mahony and the basic AHRS approach.

Table 1. RMSE values of Euler angles in degrees for Basic AHRS, Madgwick and Mahony with noise / without noise

AHRS	roll (deg)	pitch (deg)	yaw (deg)
Basic	4.3135 / 0.0353	2.0937 / 1.9871	4.4740 / 0.0000
Madgwick	0.5896 / 0.0226	0.3123 / 0.3525	0.5787 / 0.0230
Mahony	0.3813 / 0.1233	0.6464 / 0.6550	0.4522 / 0.1238

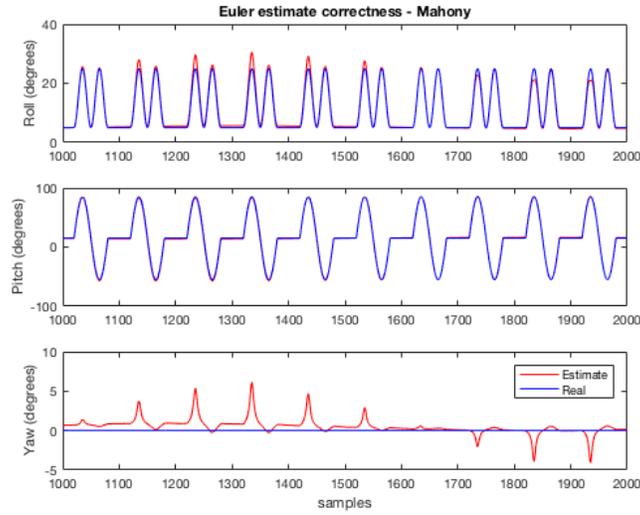


Figure 4. Euler estimation correctness of Mahony

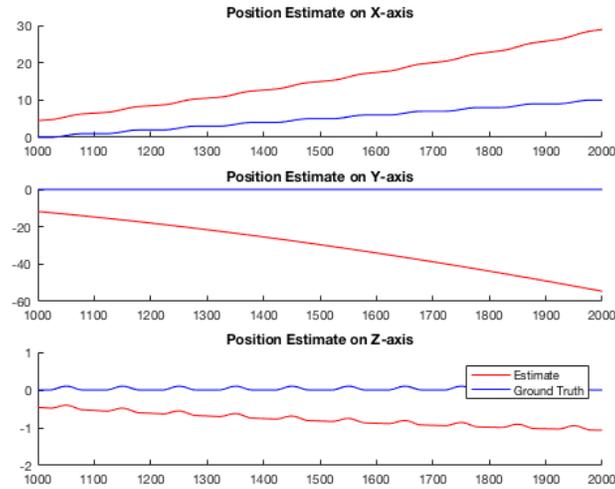


Figure 5. Position estimation of Basic AHRS

Table 2 shows the execution time or running time of the three AHRS approaches. The differences of the execution times are relatively small. Mahony has the shortest execution time of 3.1663 seconds, followed by Madgwick with 3.3308 seconds, and the basic AHRS takes 4.1083 seconds.

Table 2. Execution Time in seconds of Basic AHRS, Madgwick and Mahony

AHRS	Execution time (s)
Basic	4.1083
Madgwick	3.3308
Mahony	3.1663

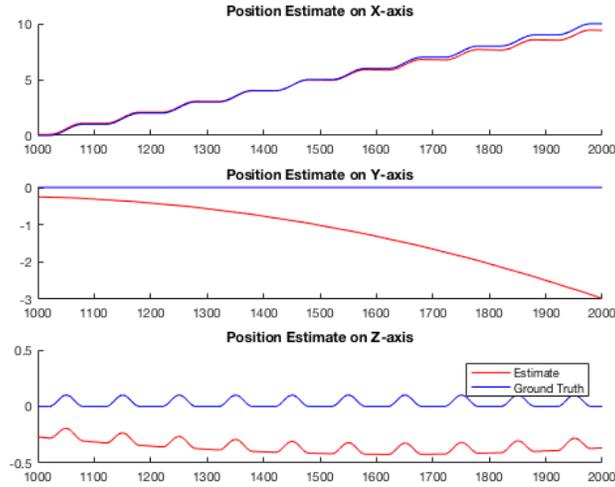


Figure 6. Position estimation of Madgwick

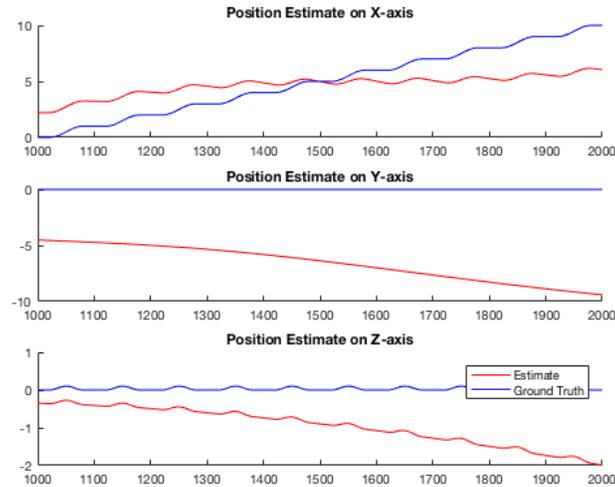


Figure 7. Position estimation of Mahony

#### 4. CONCLUSION

In order to obtain orientation information from gyroscope, accelerometer and magnetometer data a fusion algorithm has to be used. This paper compared two approaches namely Madgwick and Mahony with a basic AHRS approach. The data used was obtained from an MIMU mounted on a foot of a person. The aim was to use this motion data to estimate the Euler angles in order to find the orientation of the person. The Euler angles acquired by Madgwick, Mahony and the basic approach were then compared with the ground truth. The results show that Madgwick outperforms Mahony and the basic AHRS in terms of RMSE on the sensor data with noise when compared to the ground truth. In terms of execution time of the three approaches, Mahony takes less time to compute compared to the other two approaches.

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