Exploring worlds of more than three dimensions

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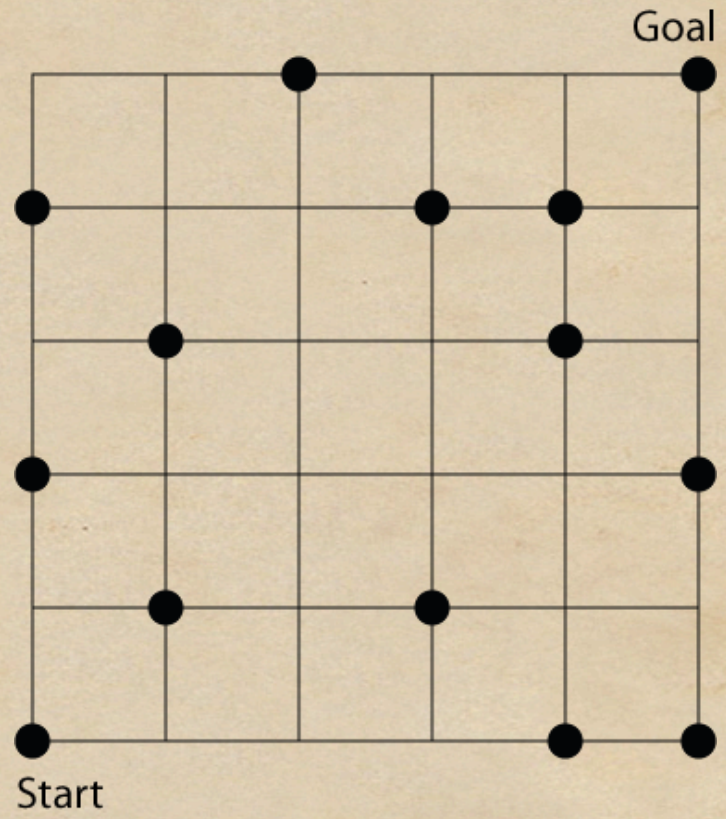
What are dimensions?

- For the purpose of this workshop
  - Number of independent attributes (coordinates)
- What does that have to do with 2-dim planes and 3-dim spaces?
  - In the plane, we need two coordinates to specify a location
  - In a 3-dim space we need three coordinates

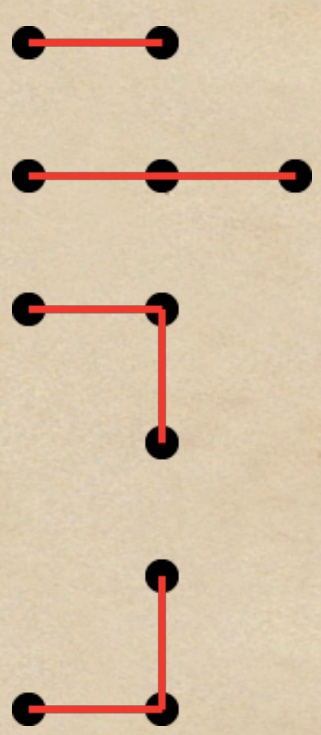
Example: 2-dim Maze Game

- Objective
  - Get from bottom left to top right
- Maximum number of steps: 2



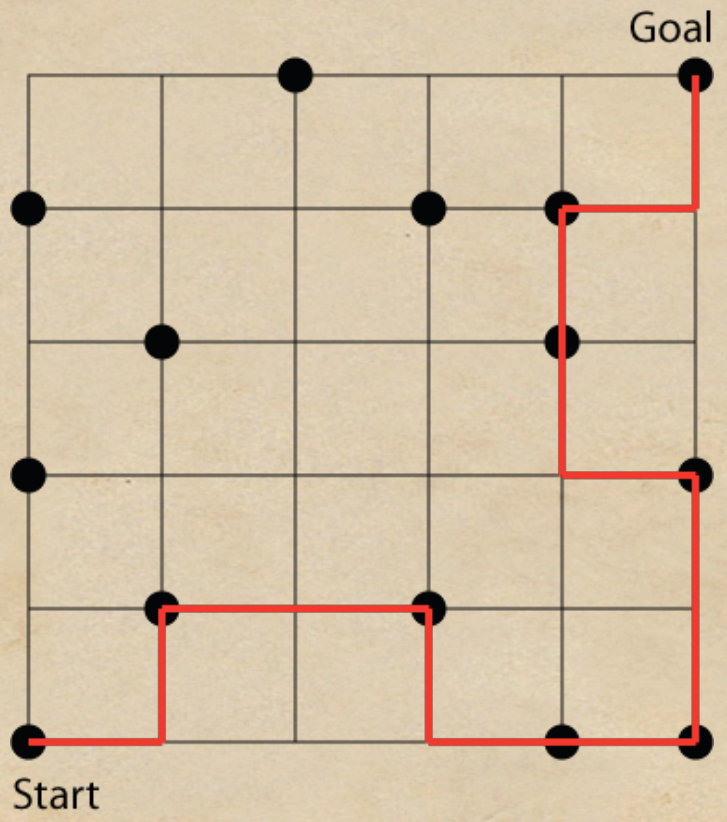
Acceptable Moves

- Move one or two steps
- You may go around corners
- No diagonals
- Fancy name:
  - Manhattan distance

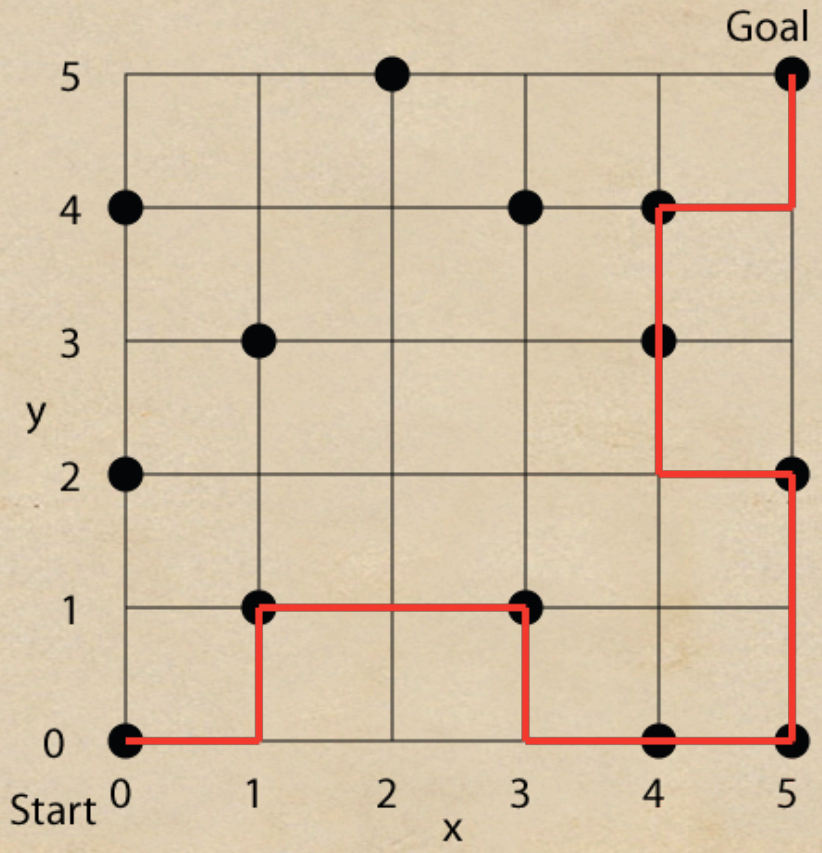


Solution to 2-dim Maze

- Possible path
- Others are possible



Maze in numbers



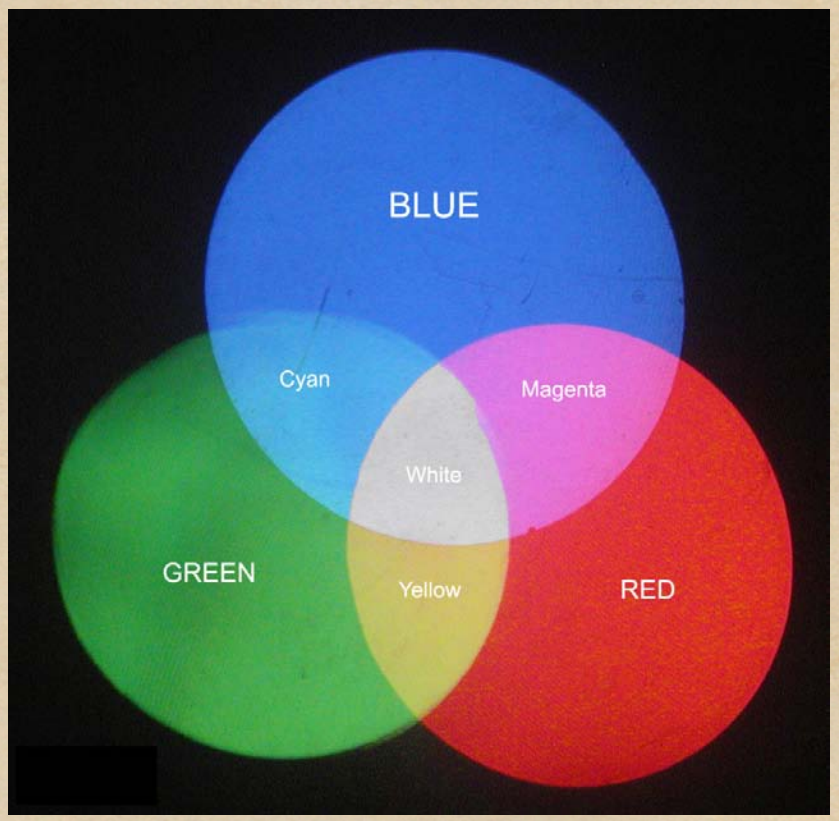
x	y
0	0
0	2
0	4
1	1
1	3
2	5
3	1
3	4
4	0
4	3
4	4
5	0
5	2
5	5

Interpretation

- What could those numbers be?
  - Geographic location
  - Age and height of a person
  - Components of color (next)
- Do not have to be what most people think when you say "dimension"

Colors

- All colors can be created by mixing 3 components
- Here colored lights are added

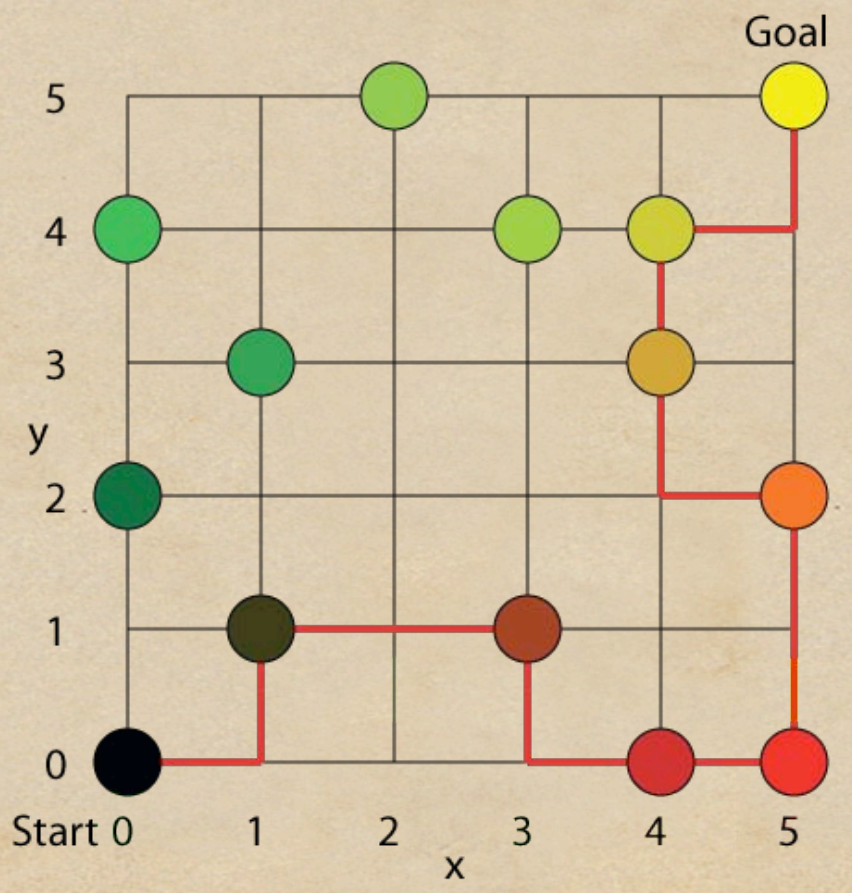


http://www.fas.harvard.edu/~scidemos/LightOptics/ColorMixing/ColorMixing06.jpg

Color Interpretation

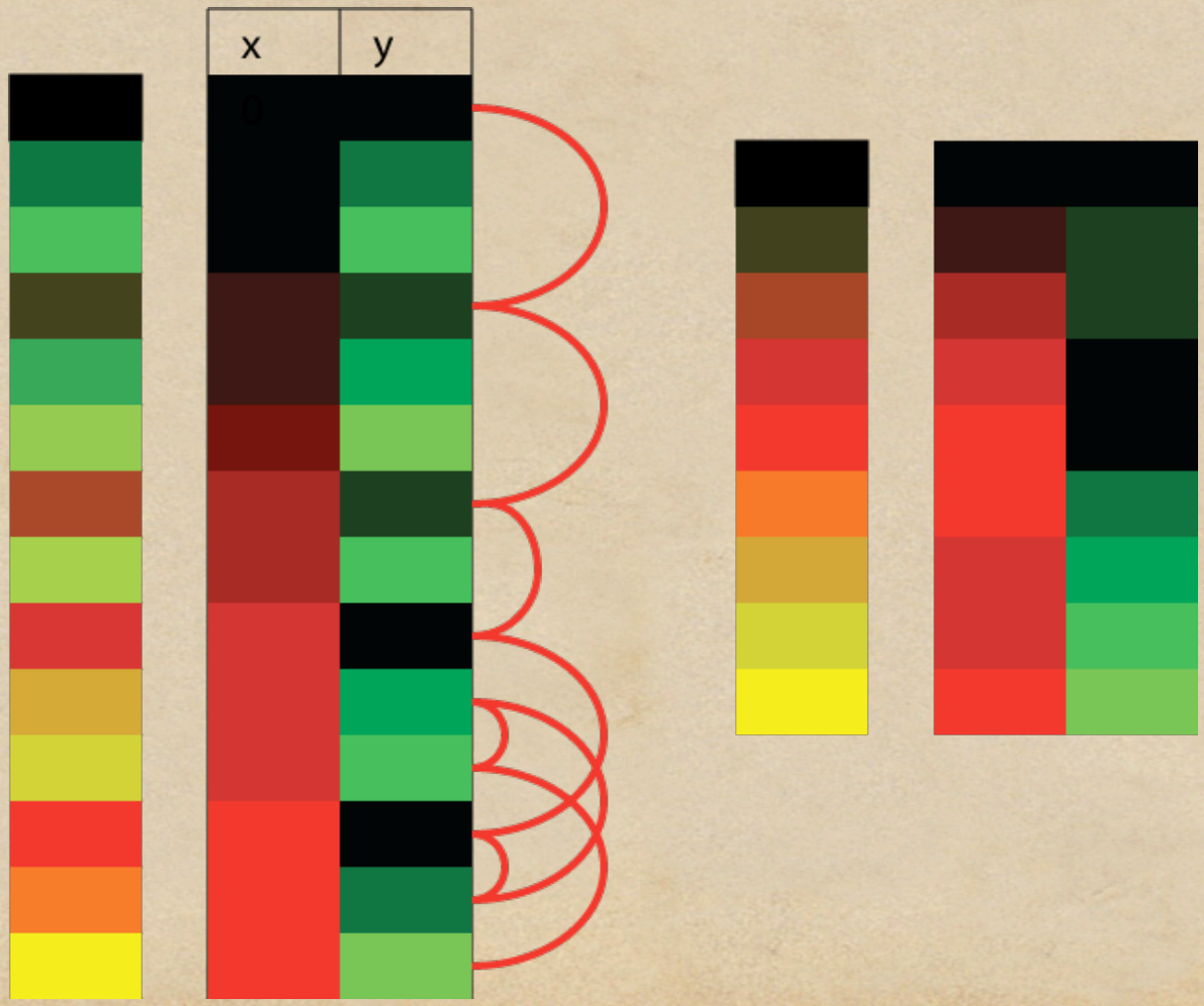
- Two basic colors
  - x-coordinate is red
  - y-coordinate is green
- Maze challenge
  - How do you find a sequence of colors with adjacent colors being "not too different"?
  - Mathematically both problems equivalent

Maze using Colors



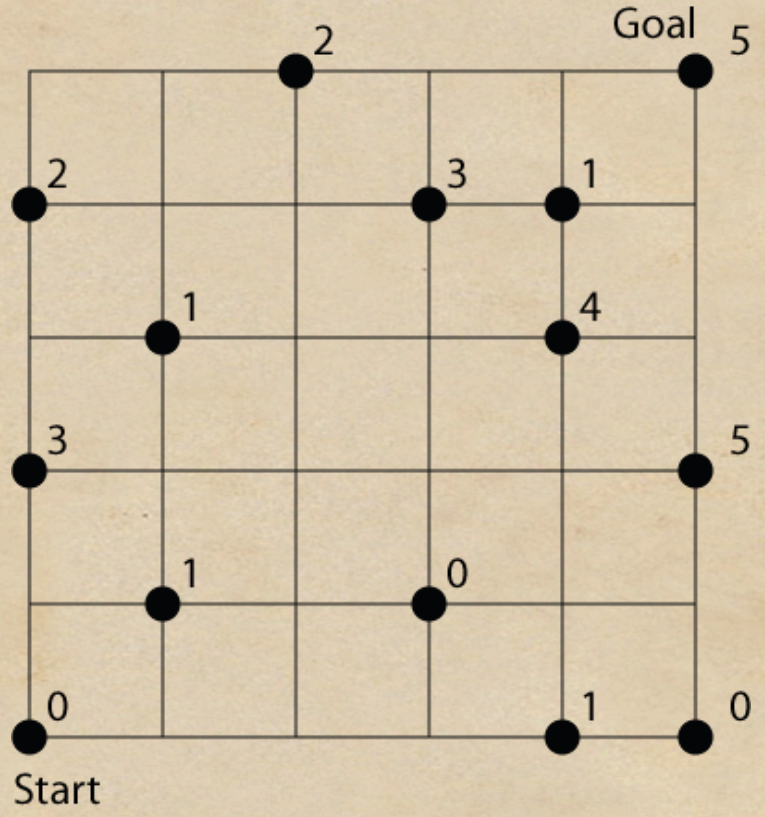
x	y
0	0
0	2
0	4
1	1
1	3
2	5
3	1
3	4
4	0
4	3
4	4
5	0
5	2
5	5

Shortest Path for 2-dim Colors



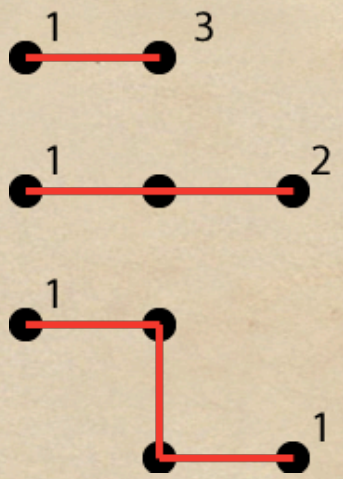
3-dim Example

- Numbers represent additional dimension
- Now 3 steps are allowed



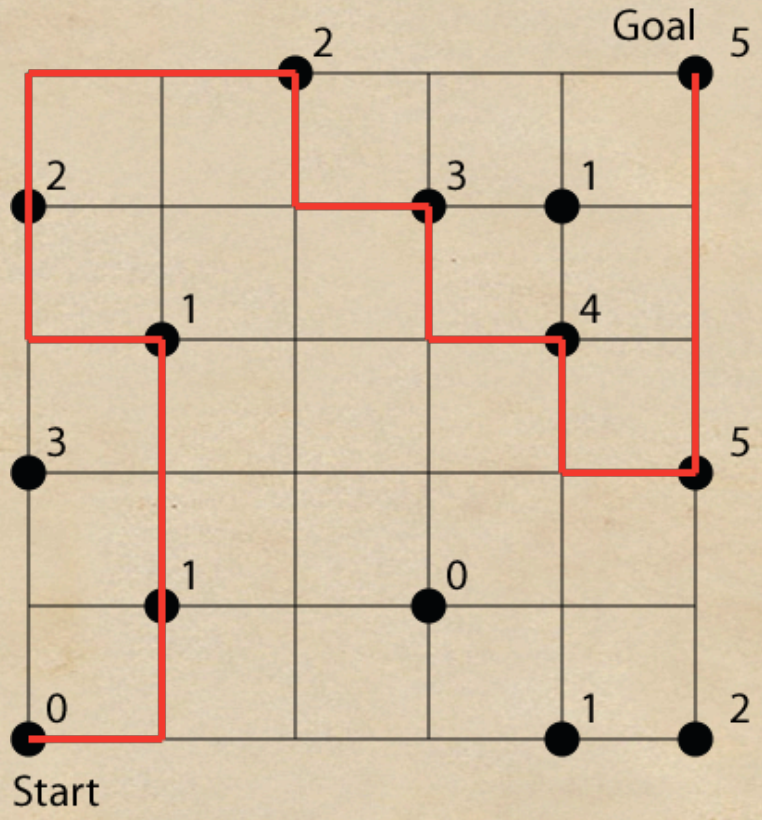
Acceptable Moves in 3-dim

- As before
  - Distance has to be smaller or equal than maximum (3)
- New
  - Difference in 3rd dimension counts as one step

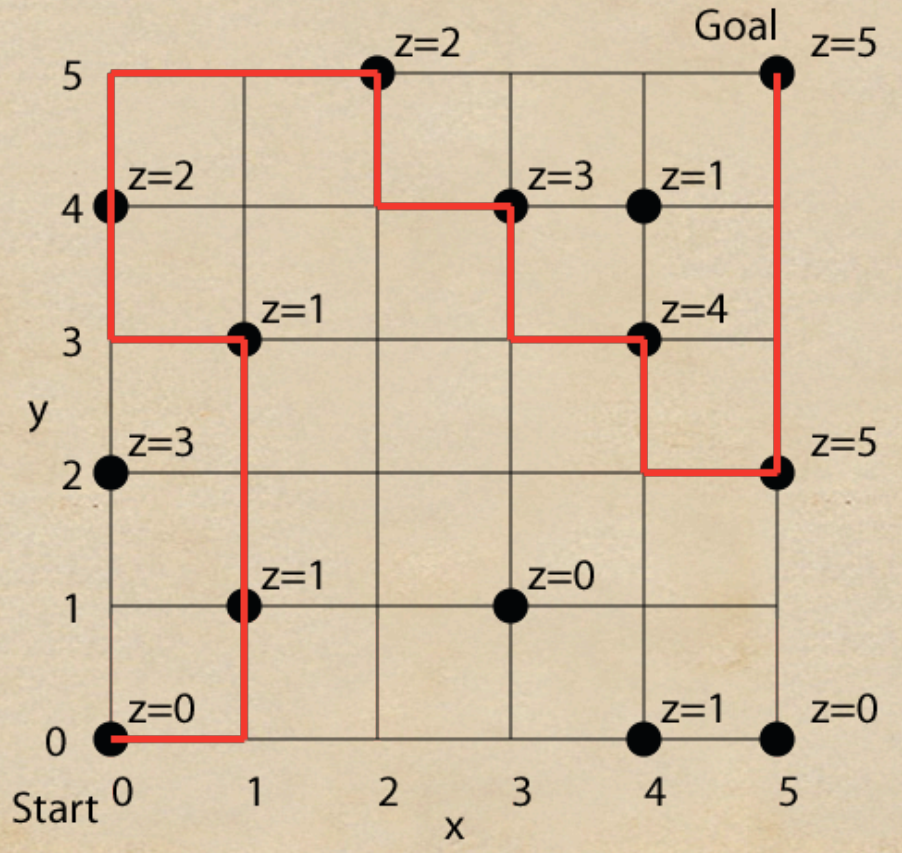


Solution to 3-dim Maze

- Note how solution differs although x and y coordinates are same

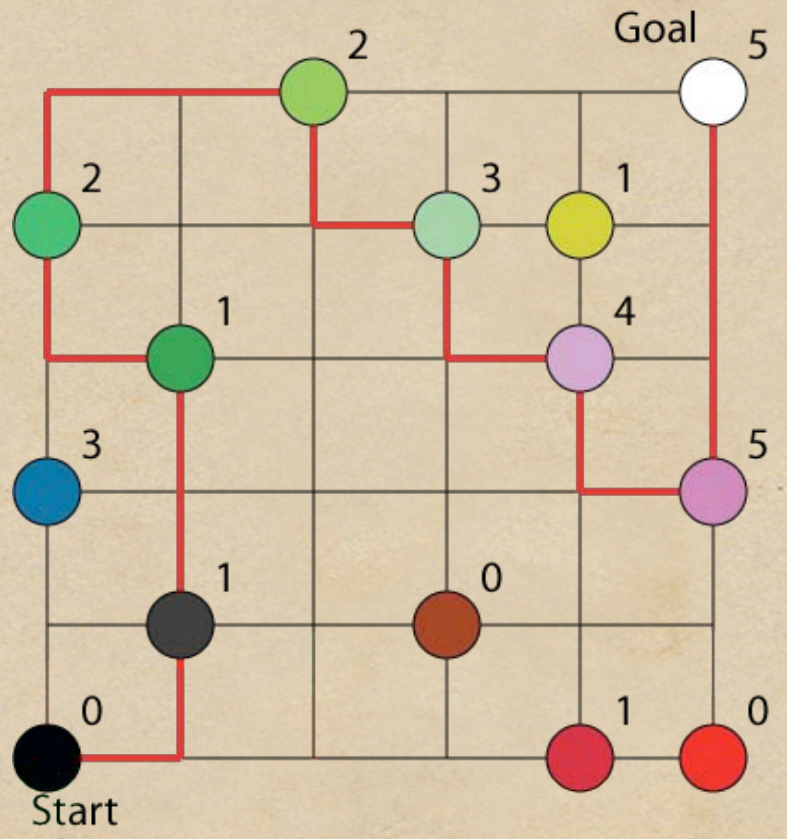


3-dim Maze in Numbers



x	y	z
0	0	0
0	2	3
0	4	2
1	1	1
1	3	1
2	5	2
3	1	0
3	4	3
4	0	1
4	3	4
4	4	1
5	0	0
5	2	5
5	5	5

3-d Maze using Color

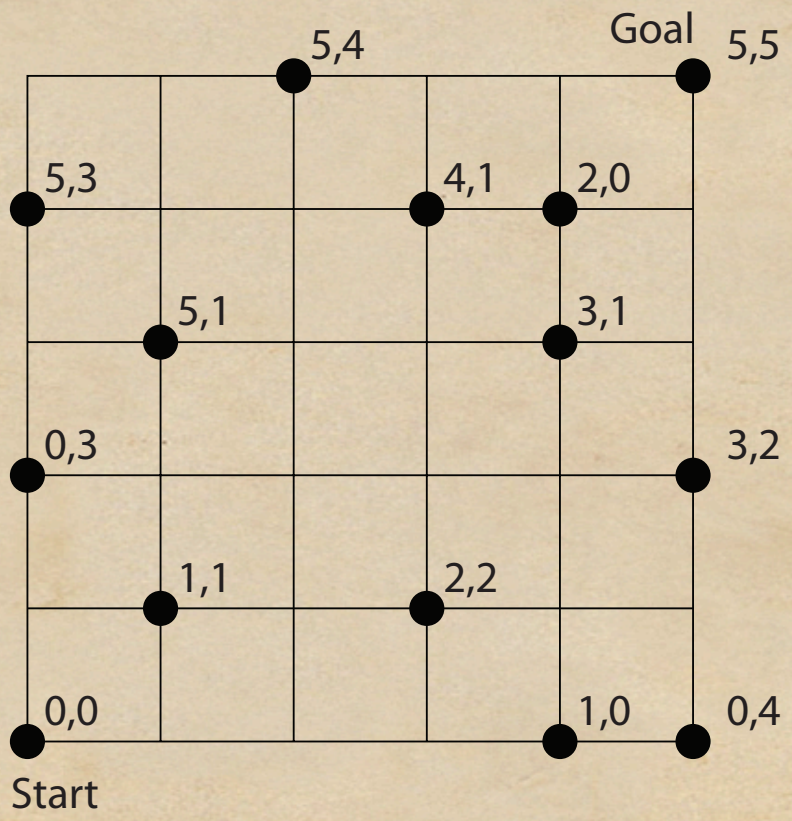


x	y	z
0	0	0
0	2	3
0	4	2
1	1	1
1	3	1
2	5	2
3	1	0
3	4	3
4	0	1
4	3	4
4	4	1
5	0	0
5	2	5
5	5	5

Shortest Path for Colors

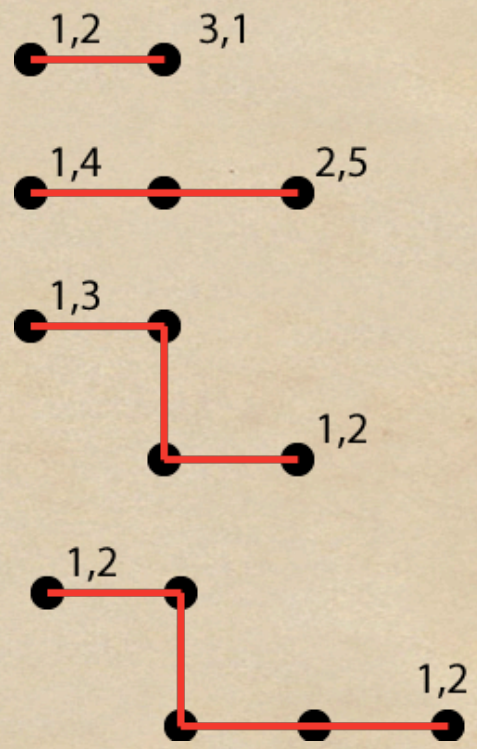
How about 4 Dimensions?

- The two numbers are independent
- Now we allow 4 steps



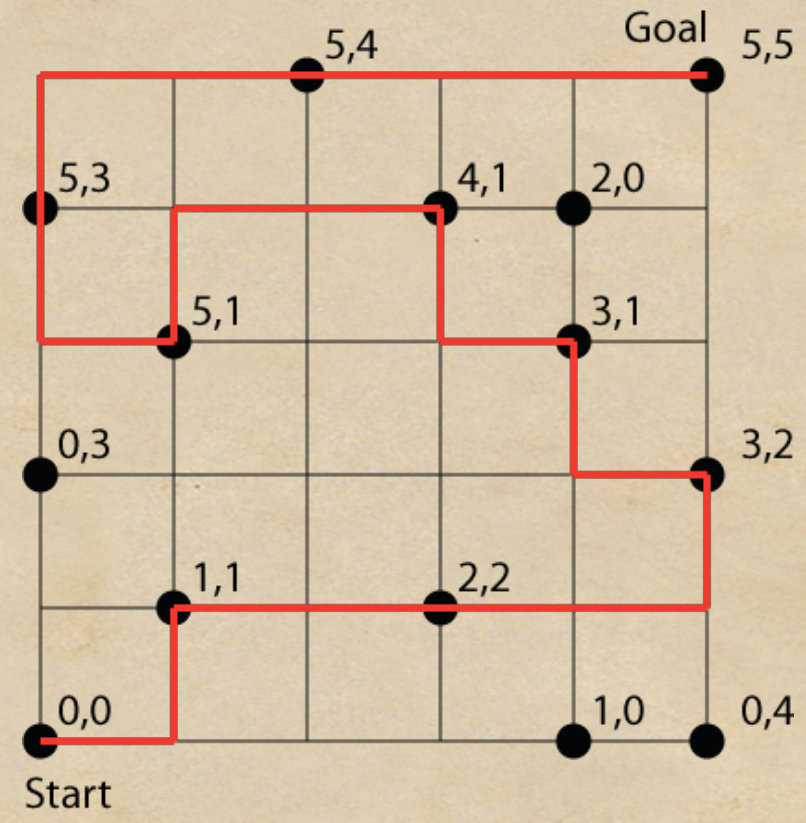
Acceptable Moves in 4-dim

- Now both additional dimensions are added to the distance
- Maximum distance is now 4 steps

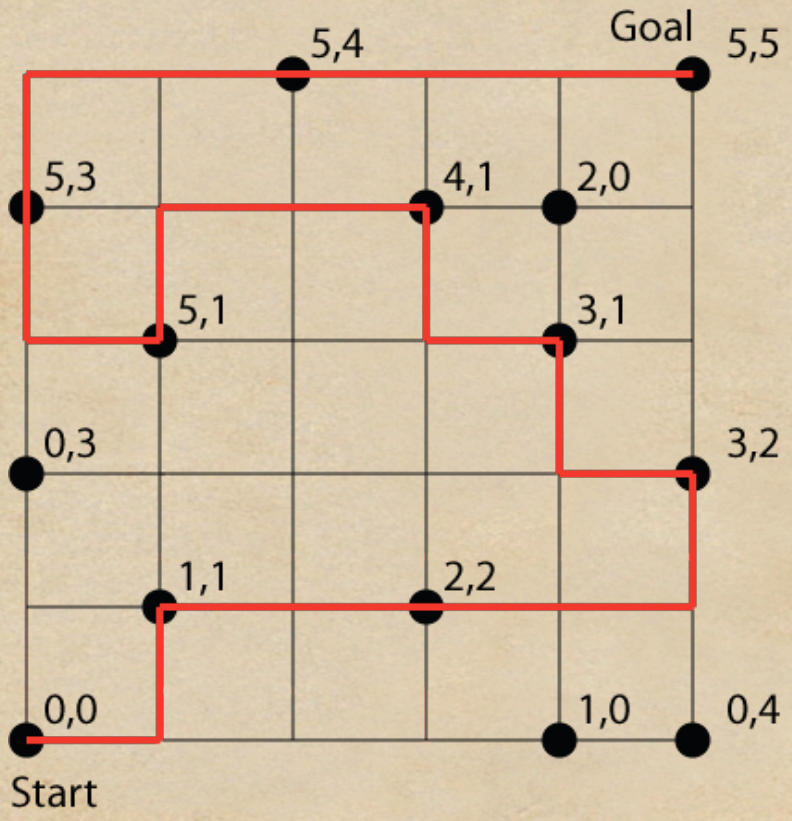


Solution to 4-dim Maze

- Note how x and y become increasingly unimportant for the overall path



4-dim Maze in Numbers



x	y	z	w
0	0	0	0
0	2	0	3
0	4	5	3
1	1	1	1
1	3	5	1
2	5	5	4
3	1	2	2
3	4	4	1
4	0	1	0
4	3	3	1
4	4	2	0
5	0	0	4
5	2	3	2
5	5	5	5

Can we do 4-dim Colors?

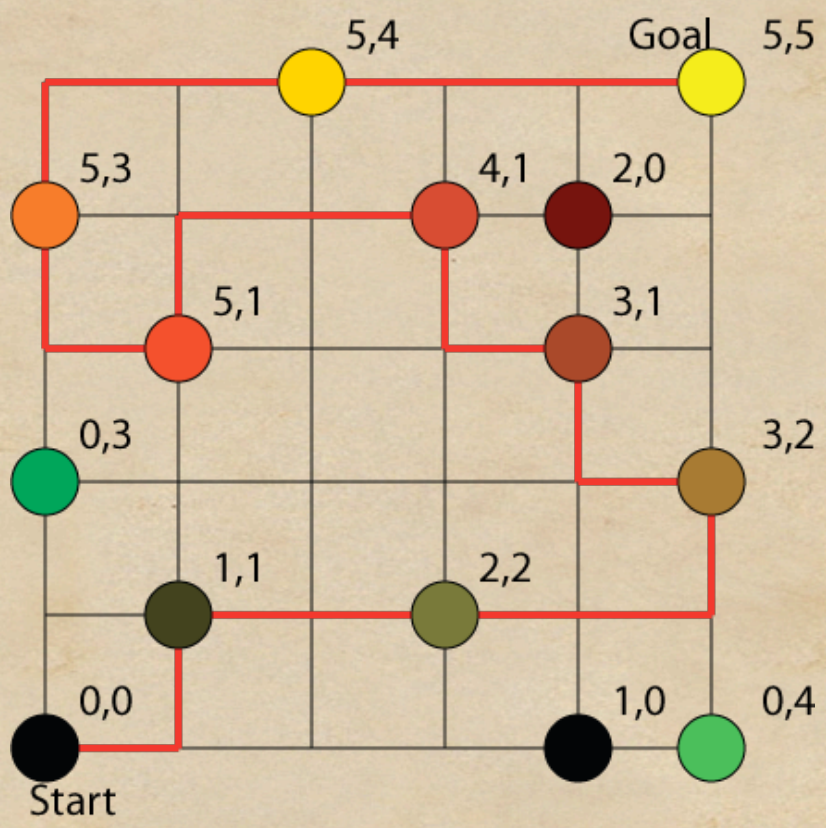
- Unfortunately human eyes only have three different receptors
  - Red, Green, and Blue
- So we can only represent 3 independent pieces of information
- Any idea how we could visualize 4 dimensions?

Combining Color and Position

- How many colors do we need to represent 4 dimensions?

Combining Color and Position

- Note how the color helps you find the path



Color and Position

- We have seen that x, y, red, and green can be used to visualize a 4-dim problem
- How many dimensions would you get by combining x and y with complete color information?
- How about complete spacial and color information?

How Common are Problems in Four and More Dimensions?

- I commonly work with 10-10,000 dimensions
- Any information that is known about a person or a thing could be another dimension
  - Some of them are yes/no types of information
  - Similar ideas and techniques
- How many things are known about you?

Suggestions Please

- What do you think is easy in high dimensions?
- What is difficult?

Some Easy Things

- Doing distance calculations
  - Not just Manhattan distance
  - Standard Euclidean distance can also easily be generalized to higher dimensions
- Calculating averages, etc.
- Many mathematical algorithms don't depend much on dimensionality

So is everything easy?

- Visualization is difficult
  - We saw that anything with more than 2 or 3 dimensions becomes difficult to visualize
- Some problems that one may not think of
  - In high dimensions there are few points very close and few points very far
  - Most are somewhere in-between

Example Application

- Understanding data
  - Grouping objects
  - Predicting something about objects
  - Finding patterns in data
- Also called: Data mining

Other Applications?

- Physical sciences / Engineering
  - Velocity components may be considered as dimensions
  - Or every position and velocity of every particle in a system may be a dimension!
- Does not end there
  - Physicists sometimes even consider infinitely many dimensions!

Summary

- Working with many dimensions is not so different from working with two or three
- There are some exciting additional challenges
- You showed that you can do it!